APPROXIMATE RELATION FOR THE HEAT FLOW TO THE WALLS OF A TUBE ARC HEATER

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In view of the increasing use being made of arc heaters ("plasmatrons") for obtaining high-temperature gas flows, the problem of the relation between the heated gas, the electric arc and the heater walls is one of considerable topical interest. Most research so far has been experimental, owing to the difficulties associated with theoretical studies, which must be based on the interrelated equations of magnetogasdynamics with various initial and boundary conditions; physicochemical changes and radiative heat transfer further complicate the structure of these equations and add to their number.

The results of attempts to establish dimensionless similarity criteria and to generalize the experimental data with the help of similarity theory [1] and dimensional analysis [2] provide a basis for the further development of modeling problems and for the representation of the experimental dimensionless characteristics of arc heaters in the form of empirical criterial relations.

There is little to be found in the literature on the theoretical determination of the characteristics of electric arc heaters [3–5] (reference [5] incorporates the results of [4]). These papers are concerned with the arc heating of a gas in a cylindrical tube of circular cross section (problem of positive arc column in a gas stream in a circular tube) in the case of laminar flow without allowance for radiation.

Apart from these studies, there have also been theoretical investigations of a cylindrical arc, in particular, the recent investigation [6] of the effect of radiation on the characteristics of an arc in argon.

In order to obtain an exact solution of arc heating problems it is obviously necessary to develop corresponding numerical methods and improve methods of determining the transport coefficients at high temperatures.

Also useful are simple approximate theoretical analyses that give an essentially correct reflection of the processes involved without yielding such exact results.

This paper presents the results of a theoretical study of the arc heating of a gas in a circular tube. An approximate relation between the heat flow to the walls and electric field intensity, tube radius, the axial value of the electrical conductivity, and the values of the heat conductivity function at the axis and the walls is given for cases of stabilized heating and absence of a flow in the tube. The accuracy of this relation is estimated on the basis of calculated and experimental data.

1. We shall consider a positive arc column in a cylindrical tube of circular cross section through which flows a gas heated by the electrical energy supplied to the arc. The outside of the tube is cooled, so that the surface washed by the hot gas has a certain definite temperature.

The approximate continuity, momentum and energy equations for steady-state laminar flow and no radiation were written for this case in [5]. The corresponding equations for turbulent flow can be obtained from these equations by the method described in [7] in connection with the plane problem of the boundary layer of a high-temperature gas. In this case the form of the equations remains the same as for laminar flow, but the coefficient of viscosity μ and the heat conductivity λ are replaced by the quantities $\varepsilon + \mu$ and $\lambda + \lambda^\circ$, where ε and λ° are the turbulent coefficient of viscosity and heat conductivity.

We shall write these equations in the form

$$\mu r \frac{\partial v_z}{\partial r} = \frac{r^2}{2}\frac{dp}{dz} - v_z \frac{\partial \psi}{\partial z} + \frac{\partial \Lambda}{\partial z}, \tag{1.1}$$

$$r\frac{\partial s}{\partial r} = -E^2\int_0^r r\sigma dr - h\frac{\partial \psi}{\partial z} + \frac{\partial \Phi}{\partial z}, \tag{1.2}$$

$$s = \int_0^T \lambda dt, \qquad \psi = \int_0^r r\rho v_z dr, \qquad -r\rho v_r = \frac{\partial \psi}{\partial z},$$

$$\Phi = \int_0^r r\rho v_z h dr, \qquad \Lambda = \int_0^r r\rho v_z^2 dr.$$

Here s is the heat conductivity function, h ethalpy, σ electrical conductivity, and ψ the stream function (the continuity equation is satisfied).

The boundary conditions are

$$v_r = 0, \quad v_z = 0, \quad h = h_w \quad \text{at} \quad r = r_w. \tag{1.3}$$

The subscripts w and 0 denote the parameters at the wall and at the axis of the tube.

Thermochemical equilibrium is assumed. Then the quantities ρ, h, μ, λ, σ for the given gas may be assumed to be known functions of temperature and pressure; these functions take into account the effects of dissociation and ionization.

In the absence of a flow ($v_Z = 0$, $\psi = \Phi = 0$) and for stabilized arc heating (ρ, v_Z, h, ψ, Φ depend only on r) the last two terms in energy equation (1.2) drop out.

If we set $r = r_W$ in (1.1), (1.2), we arrive at the integral relations of momentum and energy. We will write out the last of these:

$$2\pi r_w q_w = EI - 2\pi\left(\frac{\partial \Phi}{\partial z}\right)_w,$$

$$q_w = -r_w\left(\frac{\partial s}{\partial r}\right)_w, \qquad I = 2\pi E\int_0^{r_w}\sigma r dr. \tag{1.4}$$

№	T_0, °K	s_0 W/m	σ_0 mho/m	n	E V/m	I A	W_1 W/m	W_2 W/m	$\frac{W_1}{W_2}$	δ
1	6000	2226	74.1	2	929	10.2	9476	11330	0.836	—0.164
2	6800	3149	169.8	3	704	21.1	14850	16570	0.870	—0.130
3	8000	5120	489.8	4	462	60.0	27720	30490	0.909	—0.091
4	10000	7830	1928	4	335	122	40870	40840	1.001	0.001
5	11200	8758	3467	5	314	134	42076	39940	1.054	0.054

2. Equation (1.2) reduces to the form

$$s = s_w + \frac{E^2}{2}\int\limits_y^{y_w} \frac{dy}{y}\varphi(r, z), \quad y = \eta^2 y_w,$$

$$\eta = \frac{r}{r_w}, \quad y_w = \frac{(Er_w)^2\sigma_0}{s_0},$$

$$\varphi(r, z) = \int\limits_0^r \left[r\sigma - \frac{1}{E^2}\frac{\partial}{\partial r}\left(\frac{\partial\Phi}{\partial z} - h\frac{\partial\psi}{\partial z}\right)\right] dr =$$

$$= \int\limits_0^r r\sigma dr - \frac{1}{E^2}\left(\frac{\partial\Phi}{\partial z} - h\frac{\partial\psi}{\partial z}\right). \quad (2.1)$$

From (2.1) at y = 0 we obtain

$$s_0 = s_w + \frac{E^2}{2}\int\limits_0^{y_w} \frac{dy}{y}\varphi(r, z)$$

or, after integration by parts,

$$s_0 = s_w + \frac{E^2}{2}(\ln y_w)\left[\frac{I}{2\pi E} - \frac{1}{E^2}\left(\frac{\partial\Phi}{\partial z}\right)_w\right] - s_0\delta, \quad (2.2)$$

where

$$s_0\delta = \frac{E^2}{2}\int\limits_0^{r_w}\left[2\ln r + \ln\left(\frac{E^2\sigma_0}{s_0}\right)\right]\left[r\sigma - \frac{1}{E^2}\frac{\partial}{\partial r}\left(\frac{\partial\Phi}{\partial z} - h\frac{\partial\psi}{\partial z}\right)\right] dr.$$

On the basis of (2.2) with account for (1.4) we obtain

$$2\pi r_w q_w = \frac{2\pi(s_0 - s_w + s_0\delta)}{\ln(r_w E\sqrt{\sigma_0/s_0})}.$$

Since $\ln y = 2\ln r + \ln(E^2\sigma_0/s_0)$ changes sign in the interval of integration, conditions may arise in which the quantity $s_0\delta$ is small compared with $s_0 - s_w$. It is then possible to use the approximate relation

$$2\pi r_w q_w \approx \frac{2\pi(s_0 - s_w)}{\ln(r_w E\sqrt{\sigma_0/s_0})}. \quad (2.3)$$

In particular, this relation is applicable when there is no flow and when arc heating is stabilized. Then $2\pi r_w q_w = EI$ and, by virtue of our previous remark (paragraph 1),

$$s_0\delta = 0.5E^2\int\limits_0^{r_w} r\left[2\ln r + \ln\frac{E^2\sigma_0}{s_0}\right]\sigma dr = 0.25 s_0\int\limits_0^{y_w}\sigma^\circ \ln y dy\left(\sigma^\circ = \frac{\sigma}{\sigma_0}\right).$$

The accuracy of relation (2.3) is estimated for these cases below.

For this purpose we employed the results of calculations made together with V. I. Okrimenko in 1963. The calculations were made for air under laminar conditions (without account for radiation), taking $p = 1.0132 \cdot 10^6$ N/m², $T_w = 900°$ K, $r_w = 0.02$ m, by the method of [8]; values of λ and σ were taken from [9, 10].

Values of the quantities

$$T_0,\ s_0,\ \sigma_0,\ E,\ I,\ W_1 = EI, \quad W_2 = \frac{2\pi(s_0 - s_w)}{\ln(r_w E\sqrt{\sigma_0/s_0})},$$

$$\delta = \frac{W_1}{W_2} - 1$$

are given in the table, which also shows the number of segments n of the polygonal line approximating the curve $\sigma = \sigma(s)$.

We also used the results of calculations for argon [6] together with additional data on the dependence of I/r_w on T_0 (in the absence of radiation) at $p = 1.0132 \cdot 10^5$ N/m²; $r_w = 0.003$ m and $T_0 = 9000$–14 000° K. It was found that W_1/W_2 varies between 1.15 and 1.25.

Finally, we used experimental data on air heated by an arc in a circular tube at $r_w = 0.00317$ m, I = = 210 A [11]. In this case for stabilized heating E = = 1600 V/m the mean mass enthalpy $\langle h\rangle = 5.82 \cdot 10^7$ J/kg, and $h_0 = 2\langle h\rangle - h_w = 1.08 \cdot 10^8$ J/kg ($h_w = 8.13 \cdot 10^6$ J/kg), which corresponds (at $p = 1.0132 \cdot 10^5$ N/m²) to $T_0 = 14\,700°$ K [12]. Using the relation $\lambda = \lambda(T, p)$, $\sigma = \sigma(T, p)$, from [10, 13], we obtain $\sigma_0 = 7200$ mho/m, $s_0 = 4.47 \cdot 10^4$ W/m. From these data we find $W_1/W_2 = 0.86$.

A comparison of the value of W_2 determined from the approximate formula with W_1 determined from exact calculations and experimentally shows the applicability of Eq. (2.3) for estimating the heat flow to the heater walls.

REFERENCES

1. S. S. Kutateladze and O. I. Yas'ko, "Generalization of the characteristics of arc heaters," Inzh. fiz. zh., no. 4, 1964.

2. G. Yu. Dautov and M. F. Zhukov, "Some generalizations relating to the study of electric arcs," PMTF [Journal of Applied Mechanics and Technical Physics], no. 2, 1965.

3. G. Yu. Dautov, "Positive arc column in a flow," PMTF, no. 4, 1963.

4. J. G. Skifstad, "Analysis of the flow and heat transfer processes in a tube arc for heating a gas stream," Amer. Instit. Acronaut. Astronaut. Journ., vol. 1, no. 8, 1963.

5. J. G. Skifstad and S. N. B. Murthy, "Analysis of archeating phenomena in a tube," IEEE Trans. Nucl. Sci., vol. NS-11, no. 1, 1964.

6. V. N. Vetlutskii, A. T. Onufriev, and V. G. Sevast'yanenko, "Calculation of a wall-stabilized electric arc in argon with allowance for radiative energy transfer," PMTF [Journal of Applied Mechanics and Technical Physics], no. 4, 1965.

7. W. H. Dorrance, "Effect of dissociation on heat transfer and surface friction in a compressible turbulent boundary layer," Vopr. raketn. tekhn. [Russian translation], IL, no. 12, 1961.

8. H. Goldenberg, "Approximate solution of nonlinear differential equation giving the temperature distribution on the positive column of static arc," Brit. J. Appl. Phys., vol. 10, no. 1, 1959.

9. Yu. A. Kibardin, S. I. Kuznetsov, A. N. Lyubimov, and B. A. Shumyatskii, Atlas of Gasdynamic Functions for High-Speed High-Temperature Air Streams [in Russian], Gosenergoizdat, 1961.

10. I. R. Viegas and T. C. Peng, "Electrical conductivity of ionized air in thermodynamic equilibrium," ARS Journal, vol. 31, no. 5, 1961.

11. J. R. Jedelicka and H. A. Stine, "Axial flow through the wall-constricted direct-current arc—comparison of theory and experiment," IEEE trans. Nucl. Sci., vol. NS-11, no. 1, 1964.

12. A. S. Predvoditelev, E. V. Stupochenko, A. S. Pleshanov, E. V. Samuilov and I. B. Rozhdestvenskii, Tables of Thermodynamic Functions of Air [in Russian], Izd-vo AN SSSR, 1959.

13. F. Hansen, "Approximations for the thermodynamic and transport properties of high-temperature air," NACA Techn. Rept., NR-50, 1959.

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